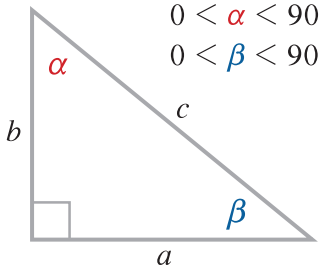
Trigonometry

Ratios and Laws

Trigonometric Ratios



$0 < \alpha < 90°$
$0 < \beta < 90°$

$\tan\alpha = \frac{a}{b}$ $\tan\beta = \frac{b}{a}$

$\sin\alpha = \frac{a}{c}$ $\sin\beta = \frac{b}{c}$

$\cos\alpha = \frac{b}{c}$ $\cos\beta = \frac{a}{c}$

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{b}{a}$ $\cot\beta = \frac{1}{\tan\beta} = \frac{a}{b}$

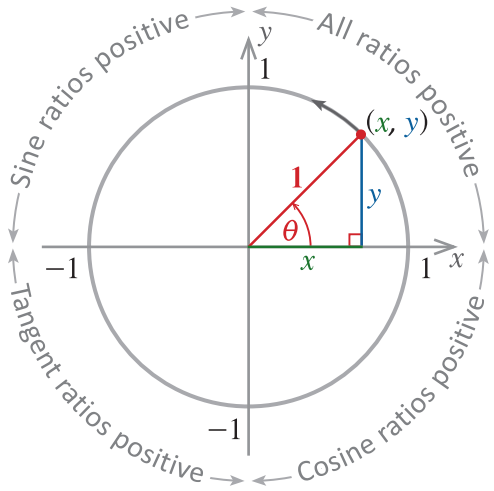
$\csc\alpha = \frac{1}{\sin\alpha} = \frac{c}{a}$ $\csc\beta = \frac{1}{\sin\beta} = \frac{c}{b}$

$\sec\alpha = \frac{1}{\cos\alpha} = \frac{c}{b}$ $\sec\beta = \frac{1}{\cos\beta} = \frac{c}{a}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

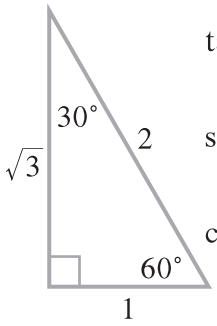
Unit Circle and Signs in Quadrants



$\tan\theta = \frac{y}{x}$ $\sin\theta = \frac{y}{1} = y$ $\cos\theta = \frac{x}{1} = x$

$\cot\theta = \frac{x}{y}$ $\csc\theta = \frac{1}{y}$ $\sec\theta = \frac{1}{x}$

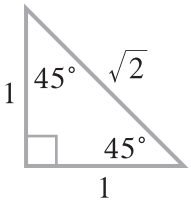
Exact Ratios



$\tan 60° = \sqrt{3}$ $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\sin 30° = \frac{1}{2}$

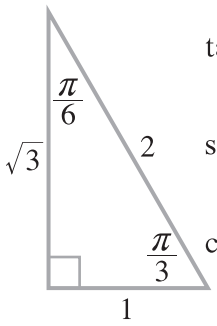
$\cos 60° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$



$\tan 45° = 1$

$\sin 45° = \frac{1}{\sqrt{2}}$

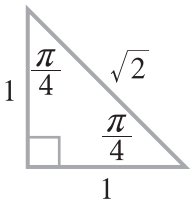
$\cos 45° = \frac{1}{\sqrt{2}}$



$\tan\frac{\pi}{3} = \sqrt{3}$ $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

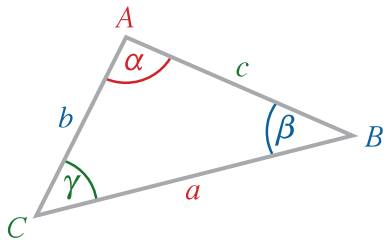


$\tan\frac{\pi}{4} = 1$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Sine Law for Triangles



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\text{Area} = \frac{1}{2}ab\sin\gamma = \frac{1}{2}ac\sin\beta = \frac{1}{2}bc\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{a\sin\beta}{b}\right) = \sin^{-1}\left(\frac{a\sin\gamma}{c}\right)$$

$$\beta = \sin^{-1}\left(\frac{b\sin\alpha}{a}\right) = \sin^{-1}\left(\frac{b\sin\gamma}{c}\right)$$

$$\gamma = \sin^{-1}\left(\frac{c\sin\alpha}{a}\right) = \sin^{-1}\left(\frac{c\sin\beta}{b}\right)$$

Cosine Law for Triangles

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \qquad \alpha = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \qquad \gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \qquad \beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

Tangent Law for Triangles

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

Mollweide's Rule

$$\frac{a+b}{c} = \frac{\cos\left[\frac{\alpha-\beta}{2}\right]}{\sin\left[\frac{\gamma}{2}\right]} \qquad \frac{a-b}{c} = \frac{\sin\left[\frac{\alpha-\beta}{2}\right]}{\cos\left[\frac{\gamma}{2}\right]}$$

Degrees and Radians

If θ is an angle in degrees and α is the same angle in radians:

$$\theta = \alpha \times \frac{180}{\pi} \qquad \alpha = \theta \times \frac{\pi}{180}$$