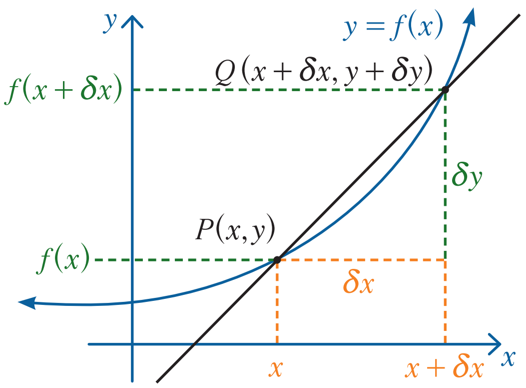
Calculus

Differentiation

Basic Differentiation



Gradient of secant PQ is:

$$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

When $\delta x = 0$, PQ is a tangent to $y = f(x)$

$$\frac{d}{dx} f(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

Basic Rules

$$\frac{d}{dx} f(x) = f'(x)$$

$$\frac{d}{dx}(ax) = a$$

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}[af(x)] = a\frac{d}{dx}[f(x)]$$

$$\frac{d}{dx}(ax^n) = nax^{n-1}$$

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1} f'(x)$$

$$\frac{d}{dx}\left[\frac{1}{x^n}\right] = -nx^{-(n+1)} = -\frac{n}{x^{n+1}}$$

More Differentiation Rules

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Product Rule

If $y = uv$, where $u = f(x)$ and $v = g(x)$

$$\frac{dy}{dx} = u'v + uv'$$

Quotient Rule

If $y = \frac{u}{v}$, where $u = f(x)$ and $v = g(x)$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

Reciprocal Rule

$$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{[f(x)]^2}$$

Absolute value

$$\frac{d}{dx}|x| = \frac{x}{|x|},\ x \neq 0$$

Second Derivative

If $f(x) = ax^n$, $f'(x) = anx^{n-1}$

$$\therefore \frac{d}{dx} f'(x) = f''(x) = an(n-1)x^{n-2}$$

Differentiation of Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = -f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\cot[f(x)]) = -f'(x)\csc^2[f(x)]$$

$$\frac{d}{dx}(\csc[f(x)]) = -f'(x)\csc[f(x)]\cot[f(x)]$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sin^{-1}[f(x)]) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$\frac{d}{dx}(\cos^{-1}[f(x)]) = \frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

Hyperbolic Functions

$$\frac{d}{dx}\sinh x = \cosh x$$

$$\frac{d}{dx}\cosh x = \sinh x$$

$$\frac{d}{dx}\tanh x = 1 - \tanh^2 x$$

$$\frac{d}{dx}\operatorname{csch} x = -\coth x \operatorname{csch} x$$

$$\frac{d}{dx}\operatorname{sech} x = -\tanh x \operatorname{sech} x$$

$$\frac{d}{dx}\coth x = 1 - \coth^2 x$$

Exponential and Logarithmic Functions

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}[a^x] = a^x \ln a$$

$$\frac{d}{dx}[a^{f(x)}] = a^{f(x)} \ln a\, f'(x)$$

$$\frac{d}{dx}[\log_e x] = \frac{d}{dx}[\ln(x)] = \frac{1}{x},\ x > 0$$

$$\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$$

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(f(x)^{g(x)}) = f(x)^{g(x)}\left(\frac{g(x)\,f'(x)}{f(x)} + \ln(f(x))\,g'(x)\right)$$